EE 362: Signals and Systems Name:----------

Spring 2019

Homework #9 (Due: Tuesday, April 2, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

1. For the circuit shown in Figure 1, find $y(t)$ if the input is [7 points]

$$x(t) = 5 + 7\cos(10\,t) + 7\sin(1000t)$$

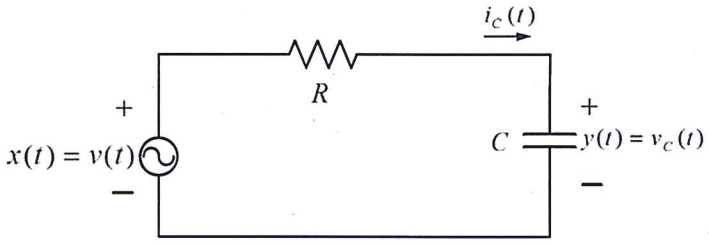


Figure 1

2. Repeat problem 1 if the input is a pulse train as shown in Figure 2 [8 points]

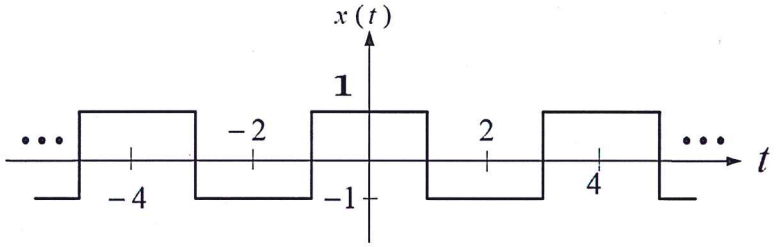


Figure 2

3. Repeat problem 1 if the output $y(t)$ is the voltage across the resistor instead of across the capacitor. [7 points]

(1)

$$H(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$y(t) = 5\,|H(0)| + 7\,|H(10)| \cos\left(10t + \angle H(10)\right) + 7\,|H(1000)| \sin\left(1000t + \angle H(1000)\right)$$

$$H(0) = 1$$

$$H(10) = \frac{1}{1 + j10RC} \Rightarrow |H(10)| = \frac{1}{\sqrt{1 + 100R^2C^2}}$$

$$\angle H(10) = 0 - \tan^{-1}(10RC)$$

$$H(1000) = \frac{1}{1 + j1000RC} \Rightarrow |H(1000)| = \frac{1}{\sqrt{1 + 10^6 R^2 C^2}}$$

$$\angle H(1000) = -\tan^{-1}(1000RC)$$

$$\therefore\ y(t) = 5 + \frac{7}{\sqrt{1 + 100R^2C^2}} \cos\left(10t - \tan^{-1}(10RC)\right) + \frac{7}{\sqrt{1 + 10^6 R^2 C^2}} \sin\left(1000t - \tan^{-1}(1000RC)\right)$$

(2)

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$$T = 4 \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2} \; ; \; a_0 = 0 \; ; \; b_k = 0$$

$$a_k = \frac{4}{T}\int_0^{T/2} x(t)\cos\left(k\frac{\pi}{2}t\right)dt = \int_0^1 \cos\left(k\frac{\pi}{2}t\right)dt - \int_1^2 \cos\left(k\frac{\pi}{2}t\right)dt$$

$$= \frac{2}{k\pi}\sin\left(k\frac{\pi}{2}t\right)\Big|_0^1 - \frac{2}{k\pi}\sin\left(k\frac{\pi}{2}t\right)\Big|_1^2$$

$$= \frac{2}{k\pi}\left[\sin\left(\frac{k\pi}{2}\right) - 0\right] - \frac{2}{k\pi}\underbrace{\left[\sin(k\pi) - \sin\left(k\frac{\pi}{2}\right)\right]}_{=0}$$

$$\therefore a_k = \frac{4}{k\pi}\sin\left(k\frac{\pi}{2}\right)$$

$$\therefore x(t) = \sum_{k=1}^{\infty} \frac{4}{k\pi}\sin\left(k\frac{\pi}{2}\right)\cos\left(k\frac{\pi}{2}t\right), \quad -\infty < t < \infty$$

$$\therefore y(t) = \sum_{k=1}^{\infty} \frac{4}{k\pi}\sin\left(k\frac{\pi}{2}\right)\left(\frac{1}{\sqrt{1+\left(k\frac{\pi}{2}RC\right)^2}}\right)\cos\left(k\frac{\pi}{2}t - \tan^{-1}\left(\frac{k\pi RC}{2}\right)\right)$$

$$-\infty < t < \infty$$

(3) $$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

$$H(0) = 0$$

$$H(10) = \frac{j10RC}{1 + j10RC}$$

$$H(1000) = \frac{j1000RC}{1 + j1000RC}$$

$$\therefore y(t) = 0 + 7\left(\frac{10RC}{\sqrt{1 + 100R^2C^2}}\right)\cos\left(10t + 90^\circ - \tan^{-1}(10RC)\right)$$

$$+ 7\left(\frac{1000RC}{\sqrt{1 + 10^6R^2C^2}}\right)\sin\left(1000t + 90^\circ - \tan^{-1}(1000RC)\right)$$